Mountain permafrost thickness evolution under influence of long-term climate fluctuations (results of numerical simulation)

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ABSTRACT: As a result of the thermal inversion, mean annual air temperatures usually decrease from the mountain slopes to the valley bottoms in East Siberia. However, the permafrost thickness below the inter-mountain depressions is typically smaller than within the rock massifs. Four major factors are responsible for permafrost thickness formation: the temperature and amplitude of long-term (100 Kyr period cycles) temperature variations at the ground surface, the relief, the geothermal heat flux variability and the vertical geocryological zonality phenomenon. A two-dimensional finite difference conductive heat transfer model with phase changes was used. The numerical modeling showed that the mountain permafrost thickness is determined by relief and also depends on climatic history. The knowledge of mountain permafrost dynamics is very important for understanding the underground water drainage and storage within the continental mountain regions.

1 INTRODUCTION

Mountain permafrost has very distinct spatial variability. The near surface ground temperature depends on landscape characteristics and on the climate variations. In addition, the mean annual permafrost temperature at 20 m depth depends on altitude (this dependence is named as “vertical or altitudinal geocryological zonality”). This vertical geocryological zonality does not always follow the vertical distribution of air temperatures (Romanovskii et al. 1991). In mountains located in maritime climates the normal geocryological zonality is typical. In that case, the mean annual temperature at 20 m depth decreases with increasing altitude. However, in Northeast Siberian mountains the inverse vertical geocryological zonality is very common. Because of the thermal inversion, the mean annual air temperatures are usually warmer on the mountain slopes than in the valley bottoms in East and Northeast Siberia. However, the permafrost is thinner below the inter-mountain depressions and river valleys than within the rock massifs. This fact was explained by the geothermal heat flux redistribution between mountain peaks and valleys (Balobayev 1991).

The authors tried to complete the sensitivity analysis and estimate the importance of four major factors, which are responsible for permafrost thickness, using numerical modeling. The knowledge of mountain permafrost dynamics is very important for understanding the underground water drainage and storage within the mountain regions, because changes in permafrost vertical and lateral distribution have a significant effect on hydrological macro conductivity and water storage capacity of rock massifs.

2 METHODS

2.1 Target setting

In the model the rock massive is presented in 2-D cross-section across the mountain peak with 9200 m width and 1800 m altitude above sea level (a.s.l.) (Fig. 1). This represents typical settings within the Udokan Range in Eastern Siberia, where the permafrost conditions were studied thoroughly (Romanovskii et al. 1989, Romanovskii et al. 1991). The steepness of slopes is 20°. The adjacent valley bottoms are at the level of 900 m altitude and their half-widths are 400 m and 3600 m correspondingly (Fig. 1). The model domain has no flux boundaries.

All four blocks are quartzitic Proterozoic sandstone (frozen thermal conductivity $\lambda_f = 3.7 \text{ W/mK}$, thawed thermal conductivity $\lambda_t = 2.9 \text{ W/mK}$, heat capacity $C_f = C_t = 2.04 \cdot 10^6 \text{ J/m}^3\text{K}$). Volumetric moisture content is 6% in blocks 2 and 4, 2% in the block number 1 and 4% in the block number 3. At the model’s lower boundary the geothermal gradient is constant

Figure 1. Chart of the model field.
where $H(t)$ expresses the energy conservation law: the Enthalpy formulation of the Stefan problem. The basic mathematical model in our approach is means that the permafrost thickness changes due to temperatures do not exceed $0°C$. It is important that during the modeling period the spatial surface temperature distribution adds to the ground surface temperature distribution, when this in case of inverse vertical zonality, and a constant ground surface temperature distribution, when this temperature does not change with elevation. The actual spatial surface temperature distribution adds to the expected periodic climate change with $100,000$-year period and with constant amplitude $5°C$ (Fig. 2).

In case of constant ground surface temperature along entire upper boundary of the calculation domain (“no vertical zonality” case), the initial surface temperature is prescribed as $-6.35°C$. In case of inverse vertical zonality this temperature is set at $-10.1°C$ at the depression and at $-5.55°C$ at the mountaintop – just opposite in comparison with normal zonality (Fig. 2). It is important that during the modeling period the temperatures do not exceed $0°C$ at the surface. That means that the permafrost thickness changes due to movements of the lower boundary of permafrost only.

2.2 Mathematical model

The basic mathematical model in our approach is the Enthalpy formulation of the Stefan problem (Alexiades & Solomon 1993, Verdi 1994). We used the quasi-linear heat conduction equation, which expresses the energy conservation law:

$$\frac{\partial H(x,y,t)}{\partial \tau} = \text{div}(k(x,y,t) \nabla t(x,y,\tau)), \ (x,y) \in \Omega,$$  

(1)

where $H(x,y,t)$ is the enthalpy

$$H(x,y,t) = \int_0^t c(x,y,s)ds + L\Theta(x,y,t)$$  

(2)

c(x, y, t) is the heat capacity, $L$ is the latent heat, $k(x, y, t)$ is thermal conductivity and $\Theta(x, y, t)$ is the volumetric unfrozen water content. We chose a smoothed step function to describe the temperature dependence of unfrozen water content. The total range of the $\Theta$ is defined by soil porosity.

The Equation (1) is complemented with boundary and initial conditions. The computational domain $\Omega = \Omega_1 \cup \Omega_2$ has a rectangular shape (Fig. 1). Homogeneous Neuman’s conditions were set at the lateral boundaries and the geothermal gradient was set at the lower boundary

$$\frac{\partial t(x,y,\tau)}{\partial y} = g, \ y = \ell_2$$  

(3)

The temperature can be specified on the upper nonlinear surface of the $\Omega_2$. However, the fictitious domain/embedding method (Marchuk et al. 1986) makes possible to “improve” the shape of the original boundary. In this method, the domain $\Omega_\epsilon$ of the original problem (unshaded region in Figure 1) is embedded into a larger computational domain, where a new artificial boundary value problem is constructed. In the new, fictitious part $\Omega_\epsilon$ of the domain (shaded region in Figure 1) the thermal conductivity coefficient becomes a vector $(k_{\text{N}}, k_{\text{P}})$. The $k_{\text{N}}$ component is chosen to be close to zero and the $k_{\text{P}}$ is chosen to be very large. Thus we make “projection” of boundary temperature from the top of $\Omega$ to non-planar ground surface. So, the Dirichlet boundary condition is imposed on the upper side of rectangular $\Omega$:

$$t_\epsilon(x,\tau) = A_\text{d}(x) + A_\text{s} \sin\left(\frac{2\pi \tau}{T}\right),$$  

(4)

where term $A_\text{d}(x)$ describes vertical zoning of the mean annual ground surface temperatures.

A fractional step approach (Godunov splitting) was used to obtain difference scheme (Marchuk 1975). The idea is to divide each time step into two steps. At each step, a different dimension is treated implicitly:

$$\frac{H(t_{n+1/2}) - H(t_{n})}{\Delta \tau_n} = 2 \left(\frac{\Delta h_{n+1,y} + \Delta h_{n,j}}{\Delta h_{n+1,y}} \right) \times \left( \frac{k_{n+1/2,y} t_{n+1/2} - t_{n+1/2} - k_{n-1/2,y} t_{n-1/2} + t_{n-1/2}}{\Delta h_{y,n}} \right),$$  

(5)

and

$$\frac{H(t_{n+1}) - H(t_{n+1/2})}{\Delta \tau_n} = 2 \left(\frac{\Delta h_{n+1,y} + \Delta h_{n,j}}{\Delta h_{n+1,y}} \right) \times \left( \frac{k_{n+1/2,y} t_{n+1/2} - t_{n+1/2} - k_{n+1/2,j} t_{n+1/2} + t_{n+1/2}}{\Delta h_{j,n}} \right),$$  

(6)

where $\Delta h_{n,x}, \Delta h_{j,y}$ are the steps of spatial no uniform grid.
The resulting system of finite difference equations is non-linear, and to solve it, the Newton’s method was employed at each time step. On the first half step (5) in case when a non-zero gradient of temperature exist we use the difference derivative of enthalpy:
\[
\frac{\partial H(t_{i,j})}{\partial t} = 0.5 \left[ \frac{H(t_{i-1,j}) - H(t_{i,j})}{(t_{i,j} - t_{i-1,j})} + \frac{H(t_{i+1,j}) - H(t_{i,j})}{(t_{i+1,j} - t_{i,j})} \right]
\]

The analytical derivative of representation (2) has to be used in case of zero-gradient temperature fields. Second half step (6) is treated similarly. Thereby, we can employ any size space grids without loss of phase transition zone for the fast temporally and spatially varying temperature fields.

3 RESULTS AND DISCUSSION

After six cycles (600,000 model years), we obtained periodically steady state temperature regime at the end of the seventh warm period. These results reflect the temperature conditions that correspond to the 50,000 years point on the time axis on Figure 2. Figure 3 presents the temperature field patterns without vertical geocryological zonality (a), with normal vertical zonality (b) and with inverse vertical zonality (c).

The 2-D simulation shows that the mountain relief affects the temperature field at the depth as deep as 3000 m. Even at depths 2000–2500 m we can see the influence of vertical temperature zonality on the temperature field. The inverse type of vertical zonality provides the anomalous thickness in wide depressions and the increased permafrost thickness under mountain peaks.

The extent of cooling of valleys during the cold part of the cycle depends on width of the valleys. In case of normal vertical zonality the small valley cools deeper than the large one. In case of inverse vertical zonality we have an opposite situation.

We use the method of heat flow lines visualization for understanding the degree of redistribution of heat fluxes within relief and the effects of vertical zonality on this redistribution. The normal vertical zonality practically nullifies the effect of heat flux redistribution under mountain relief. On the contrary, the inverse vertical zonality favours this redistribution (Fig. 4). Maximum concentration of streamlines can be seen at

Figure 3. Distribution of the simulated temperature at the end of the warm period: a-absence of the permafrost altitudinal zonality, b-normal (marine) zonality, c-inverse zonality. X-axis represents distance (m), Y-axis represents depth (m).

Figure 4. a – Lines of geothermal heat fluxes at the end of a warm period, no altitudinal permafrost temperature zonality, b – the same case with normal (maritime) vertical zonality, c – the same case with inverse vertical zonality. X-axis represents distance (m), Y-axis represents depth (m).
the foot of mountain peak. This fact accords with the observed local warm zone along the lower part of slopes.

The temperature field dynamics under mountain relief is shown in Figure 5. If the surface temperature does not change with altitude then the delay of permafrost bottom maximum thawing under the mountain peak is about 10,000 years after the warming maximum. The corresponding freezing delay at the permafrost base is 15,000 years. Therefore the bottom thawing under the mountain peak is faster than bottom freezing. The permafrost thickness varies under mountain from 800 to 1500 m (Fig. 5a). Under the middle part of the large valley the corresponding periods are 12,000 and 18,000 years; the permafrost thickness varies under valleys from 400 to 1000 m (Fig. 5b).

In case of normal vertical zonality the lag of permafrost thickness reduction in comparison with climatic maximum is 14,000 years; the lag of permafrost thickness augmentation in comparison with climatic minimum is 17,000 years.

The permafrost thickness varies under the mountain from 800 to 1500 m (Fig. 5c). Under the middle part of the large valley the corresponding periods are 10,000 and 15,000 years; the permafrost thickness varies under the valley from 300 to 900 m (Fig. 5d).

In the case of inverse type of vertical zonality the lag of permafrost thickness reduction under mountain in comparison with climatic maximum is 15,000 years and the freezing delay is 17,000 years. The permafrost thickness varies under the mountain from 1200 to 1900 m (Fig. 5e). Under the middle part of the large valley, the corresponding periods are 16,000 and 20,000 years. The permafrost thickness varies under the valley from 950 to 1300 m (Fig. 5f).

Evidently, the continental type of climate that leads to winter temperature inversion contributes to forming the largest thickness of permafrost that is also the most mobile (Table 1).
4 CONCLUSIONS

The developed numerical model promotes the possibility to assess the 2-D permafrost dynamics under the periodic climate change conditions, for different topographies and different types of altitudinal ground surface temperature zonality.

A periodically steady-state temperature regime results in different permafrost thickness under mountains and valleys due to heat flux redistribution by mountain relief. The mountain relief affects the temperature field patterns to the depth of 3000 m.

The inverse type of vertical geocryological zonality results in an anomalous thickness in wide depressions together with the increase in permafrost thickness under mountain peaks.

The model can be used in the future to estimate the influence of different types of relief and ground surface temperature changes (deterministic or stochastic) on permafrost thickness dynamics.

REFERENCES


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