Introduction

An artificial neural network (ANN) can be a powerful tool for modeling a system while having incomplete or a little understanding of its governing laws, or for modeling a system in which the governing laws are too complex to be solved. A remarkable feature of a neural network is that it can learn from the experience of the relationship between the input and output given by the experiments, and it is capable of generalizations based on the knowledge obtained.

The application of ANN for modeling geotechnical problems has been investigated by several researchers. Ghaboussi et al. (1991) used a Back-propagation (BP) network (Rumelhart et al., 1986) for modeling soil stress-strain relations. Goh (1995) also applied a BP network for an interacting system, modeling cone penetration and the predicted load capacity of driven piles. A BP network is very commonly used in neural network applications because of its simple learning algorithm, but it has a low efficiency of learning. In addition, the design of hidden layers and hidden neurons in the BP network largely relies on the researcher’s experience.

By comparison, a Radial basis function (RBF) neural network has a high learning efficiency in training the network. The structure of an RBF network has self-organizing characteristics, which allow for adaptive determination of the hidden neurons in the training network. Recently, Flood (1996) has demonstrated a method for simulating the partial differential equation of heat flow in soil by an RBF network. Kushwaha and Zhang (1997) also employed an RBF network to describe a soil-tool interacting system with a multi-input and multi-objective problem.

The objective of this paper is to model soil temperature over time during soil freezing and thawing. In addition, the relationship between the frozen soil strength and the confining pressure, strain rate, and soil temperature is simulated.

Radial basis function neural networks

An RBF neural network has a feed-forward architecture, consisting of only one hidden layer together with one input layer and one output layer, as shown in Figure 1. Each input neuron is connected with all hidden neurons, and hidden neurons and output neurons are also interconnected with each other by weight $w$.

The output response of hidden neurons has the form of a radial basis function. A Gaussian function (kernel) is the most widely employed for each hidden neuron’s response, which can be expressed as

$$ h_j = \exp \left( -\frac{\|x - c_j\|^2}{2\sigma_j^2} \right) \quad j = 1, 2, ..., J \quad [1] $$

Abstract

A radial basis function (RBF) neural network was applied to simulate soil freezing and thawing processes. The air temperature, soil depth, and time were used as inputs to the neural network, and soil temperature was the output. The relationship between soil temperature, air temperature, elapsed time, and depth was learned from the experimental data by training the RBF network. The trained network can predict soil temperature for new inputs of depth, air temperature, and time. In a second example, the RBF network was employed to simulate the dependency of frozen soil strength on confining pressure, strain rate, and soil temperature. By comparing with experimental data, it was shown that the network can yield a very satisfactory generalization of the frozen soil strength in relation to these variables.

Z.X. Zhang¹, R.L. Kushwaha²

Department of Agricultural and Bioresource Engineering
University of Saskatchewan, Saskatoon, SK. Canada S7N 5A9
1. e-mail: zhangz@engr.usask.ca
2. e-mail: kushwaha@engr.usask.ca

SIMULATION OF FREEZING AND FROZEN SOIL BEHAVIOURS USING A RADIAL BASIS FUNCTION NEURAL NETWORK

Z.X. Zhang¹, R.L. Kushwaha²

Department of Agricultural and Bioresource Engineering
University of Saskatchewan, Saskatoon, SK. Canada S7N 5A9
1. e-mail: zhangz@engr.usask.ca
2. e-mail: kushwaha@engr.usask.ca
Where
\[ h_j \] is the output of hidden neuron \( j \),
\( x \) is the vector of input pattern,
\( c_j \) is the center of hidden neuron \( j \),
\( \sigma_j \) is the width of hidden neuron \( j \)
\[ \| \cdot \| \] is the Euclidean distance,

The response of the Gaussian function has a maximum output at the center, and a rapid decay to zero as the input signals become more distant from the center of the receiving field. Those signals that are at the center of the receiving field are called prototypes. The hidden neurons of an RBF network have a property that only responds to the input signals that are close to the prototypes. The characteristic that neurons have such a locally tuned, or selective response to the receiving field, can be found in some nervous systems (Moody and Darken, 1989). It is evident that the response domain of Gaussian neurons from the center is related to the width, \( \sigma_j \). In the real world, it is often assumed that nearby patterns are more likely to have the same classification than are distant ones, and that similar things tend to have the same properties (Anderson, 1995). Therefore, the Gaussian kernel has a property of responding only to similar classifications.

The neurons in the output layer have a linear output response to the signals that are transmitted from the hidden neurons. For example, the output of neuron \( k \) in the output layer can be expressed as

\[
y_k = \sum_{j=1}^{J} w_{kj} h_j \quad k = 1,2,\ldots,K
\]  

[2]

where \( w_{kj} \) is the weight connected to hidden neuron \( j \) and output neuron \( k \).

From the surface modeling interpretation, Eq. (2) can be interpreted as a hypersurface with the superposition of Gaussian bell-shaped surfaces. The shape of the hypersurface depends on the number of hidden units, and the location in input space, width and weight. The task of training the RBF network is to determine the value of the center, width and weight so as to make the output hypersurface fit the target surface. Thus, the training of the RBF networks can be performed by a strategy in which the hidden neurons are trained one by one. In training every hidden neuron, the location of the center is assigned to the training pattern in input space where the peak of the sum of squared error occurs. For this purpose, the k-mean clustering technique was employed (Moody et al., 1989). The width is set as a global constant in this paper (i.e. the width of all hidden neurons is a constant with the same value) but it was found that the capacity of the RBF network generalizations was closely dependent on the width.

**Simulation of ground temperature during freezing and thawing**

Theoretically, the distribution of ground temperature can be computed using numerical methods such as finite element or finite difference methods, based on soil and water conduction. However, it is difficult to acquire full information on the physical and thermal properties related to the surface radiation balance, and soil and water heat flow. From the point of view of system analysis and engineering application, however, it would be useful if ground freezing and thawing processes could be described by macro-observational factors that can be easily measured, such as air temperature. For a given location, soil temperature is associated with air temperature, elapsed time, and the depth below the ground surface. It could be simulated with a black or grey box, as shown in Figure 2.

The relationship between the system input and output in Figure 2 can be learned from the measured data by training a network. Based on the experience that has been gained, the trained network will make a reasonable generalization for the new input information.

To illustrate this argument, an experiment on soil freezing and thawing process was conducted in the laboratory. Measured air temperature in a cold cabinet
and soil temperature at different depths are shown in Figure 3. Twenty six data points for air temperature and elapsed time were extracted from these measured data for training the network (Table 1). From these values, a training pattern, which was composed of 156 learning samples, was constructed such that

\[ P_{i} = \left[ d_{i} \theta_{ai} t_{i} \right]^{T} \]

where \( P \) and \( T \) are the input and target output of training pattern, respectively,

\[ P_{i} = [d_{i} \theta_{ai} t_{i}]^{T} (i = 1, 2, \ldots 156) \]

\[ d_{i}, \theta_{ai} \] and \( t_{i} \) are depth, air temperature, and elapsed time for \( i \)th learning sample, respectively,

\( T \) is the transition of matrix,

![Figure 3. Air and soil temperatures vs. elapsed time.](image)

![Figure 4. Squared-sum-error vs. CPU time.](image)
Experience indicated that normalization of the input and target pattern, $P$ and $T$, improves the learning efficiency in training a network. For this purpose, a transformation is made as follows:

$$
P = \begin{bmatrix} P_1 & P_2 & \cdots & P_{40} \end{bmatrix}
$$

$$
T = \begin{bmatrix} q_1 & q_2 & \cdots & q_{40} \end{bmatrix}
$$

where $d_{\text{max}}$, $t_{\text{max}}$, $\theta_{\text{am}}, \theta_{\text{sm}}$, and $a$ are maximum values of depth, elapsed time, absolute value of air temperature and soil temperature, respectively.

According to the given training pattern, and in case of setting $\sigma = 3$, 50 hidden neurons were trained. It took about 84 s of CPU time for a computer with chip frequency of 15 MHz, and 8 MB of RAM before a target squared-sum-error of 0.02 had been reached (Figure 4).

To test the capability of generalizations for the network that has been trained, a test input pattern was designed, in which the depth changed from 0 to 500 mm with an increment of 10 mm, and the value of air temperature and elapsed time are given in Table 2. The trained network predictions as a function of depth at different elapsed times are shown in Figure 5a for freezing and Figure 5b for thawing. It can be seen from these figures that the trained network yields a good generalization for the given test pattern, except for a few cases. For example, at times of 178 hr during soil freezing and 880 hr during soil thawing, the network predictions, to some degree, deviated from the measured value. The error of prediction compared to the measured value is also given in Appendix.

Because of the locally tuned characteristics of the RBF hidden neurons, there is a lower response of Gaussian neurons to the input signals that are located at the edge of the input field. Particularly, when a new input signal is beyond the domain of input field of the training pattern, the RBF network generates a bigger error of generalization. In other words, the RBF network is at generalization during interpolation, but is weak at extrapolation. Generally speaking, it is a hard task to make good generalizations in the sense of extrapolation, even for human beings. Therefore, it is important for practical applications to construct a proper training pattern, such that the domain of input space chosen is as large as possible to improve the performance of network prediction.

### Simulation of frozen strength

The strength of frozen soil is closely associated with temperature, strain rate (or loading time), and confi-
ning pressure, in addition to soil type, soil density, and soil moisture. Up to now, no well-established theory can describe the behaviours of frozen soil strength related to a change in these factors. Since limited data is available, two simple examples were used to depict the behaviours of frozen soil strength via a neural network.

Figure 6 shows frozen sand strength in relation to confining pressure and soil density and saturation. The mechanical and thermal effects of ice could account for the characteristics of frozen soil strength relating to confining pressure and dry density (Zhang et al., 1993). To simulate the effect of soil dry density on the relationship between frozen soil strength and confining pressure using an RBF network, dry density and confining pressure were used as the network inputs, and frozen soil strength as output. In this example, a total of 40 learning samples were chosen for constructing the training pattern, that is

\[ P = \begin{bmatrix} P_1 P_2 \ldots P_{40} \end{bmatrix} \tag{5} \]

\[ T = \begin{bmatrix} q_1 q_2 \ldots q_{40} \end{bmatrix} \]

where

- \( P \) is the input training pattern,
- \( P_i = [\gamma_i \rho_i]^T \) (i = 1, 2, ..., 40)
- \( \gamma_i \) and \( \rho_i \) are soil dry density and confining pressure for \( i \)th learning sample, respectively,
- \( T \) is the transition of matrix,
- \( T \) is the target output of training pattern,
- \( q_i \) is the maximum deviatoric stress of frozen sand corresponding to the \( i \)th input learning sample.

After the network was trained, the network predictions as a function of confining pressure for different dry densities are illustrated as solid lines in Figure 6. Although there are no experimental data available for verification of the network predictions at dry densities of 1700 kg/m\(^3\) and 1800 kg/m\(^3\), the predicted curves display a reasonable tendency with respect to the measured data at soil densities of 1600 kg/m\(^3\) and 1900 kg/m\(^3\).

Similarly, the results of an RBF network modeling of frozen silt strength with change in strain rate and temperature are given in Figure 7. In this example, the strain rate and temperature acted as input to the network, and the compressive strength of frozen silt was the output. The soil temperatures at -1\(^\circ\)C, -5\(^\circ\)C, and -10\(^\circ\)C were chosen as learning samples for training network, as shown by solid symbols in Figure 7. The training pattern is given as

\[ P = \begin{bmatrix} P_1 P_2 \ldots P_{70} \end{bmatrix} \]

\[ T = \begin{bmatrix} \sigma_1 \sigma_2 \ldots \sigma_{70} \end{bmatrix} \tag{6} \]

Z.X. Zhang, R.L. Kushwaha
where,

\[ P_i = [\theta_i \, e_i]^T \quad (i = 1, 2, ..., 70) \]

\( \theta_i \) and \( e_i \) are soil temperature and strain rate for \( i \)th learning sample, respectively,

\( \sigma_i \) (\( i = 1, 2, ..., 70 \)), is the uniaxial compressive strength of frozen silt corresponding to the \( i \)th learning sample.

After the network training was completed for the target of squared-sum-error of 0.02, only 18 hidden neurons were trained. The trained network made satisfactory generalizations for the new input pattern in which strain rate changed from \( 10^{-7} \, \text{s}^{-1} \) to \( 1 \, \text{s}^{-1} \), and the temperature varied from \(-0.5^\circ\text{C}\) to \(-10^\circ\text{C}\) (Figure 7).

### Conclusion

An RBF network is an effective tool for recognition of a system pattern whose governing laws have not been fully understood, if there is plenty of experimental data available for system simulation. The system model established by a neural network is equivalent to an empirical model, but the benefits of ANN simulation over statistical analysis is that it can establish highly nonlinear relations to the multivariable inputs. The examples discussed in this study demonstrate that an RBF network can give very satisfactory generalizations in the sense of interpolation. It is anticipated that ANN could be utilized for modeling the creep deformation of frozen soil and ice. Finally, it must be noted that because of the locally tuned feature of Gaussian neuron response, an RBF network is weak in extrapolation. To improve this shortcoming, the domain of input space chosen should be as large as possible.

### Acknowledgments

The authors are grateful for the financial support from the National Science and Engineering Research Council of Canada.

### References


### Appendix

#### Measured values of soil temperature

<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>6.9</td>
</tr>
<tr>
<td>400</td>
<td>5.6</td>
</tr>
<tr>
<td>300</td>
<td>4.3</td>
</tr>
<tr>
<td>200</td>
<td>3.3</td>
</tr>
<tr>
<td>100</td>
<td>2.5</td>
</tr>
<tr>
<td>0</td>
<td>-0.6</td>
</tr>
<tr>
<td>500</td>
<td>4.9</td>
</tr>
<tr>
<td>400</td>
<td>3.9</td>
</tr>
<tr>
<td>300</td>
<td>2.8</td>
</tr>
<tr>
<td>200</td>
<td>2.8</td>
</tr>
<tr>
<td>100</td>
<td>2.2</td>
</tr>
<tr>
<td>0</td>
<td>-1.1</td>
</tr>
<tr>
<td>500</td>
<td>2.6</td>
</tr>
<tr>
<td>400</td>
<td>1.9</td>
</tr>
<tr>
<td>300</td>
<td>1.1</td>
</tr>
<tr>
<td>200</td>
<td>0.6</td>
</tr>
<tr>
<td>100</td>
<td>0.1</td>
</tr>
<tr>
<td>0</td>
<td>-3.4</td>
</tr>
</tbody>
</table>

#### Predicted values of soil temperature

<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.57</td>
</tr>
<tr>
<td>400</td>
<td>0.54</td>
</tr>
<tr>
<td>300</td>
<td>0.51</td>
</tr>
<tr>
<td>200</td>
<td>0.48</td>
</tr>
<tr>
<td>100</td>
<td>0.47</td>
</tr>
<tr>
<td>0</td>
<td>0.47</td>
</tr>
</tbody>
</table>

#### Absolute error (measured value - prediction)

<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>Absolute error</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>-0.07</td>
</tr>
<tr>
<td>400</td>
<td>-0.14</td>
</tr>
<tr>
<td>300</td>
<td>-0.12</td>
</tr>
<tr>
<td>200</td>
<td>-0.39</td>
</tr>
<tr>
<td>100</td>
<td>-0.07</td>
</tr>
<tr>
<td>0</td>
<td>-0.07</td>
</tr>
<tr>
<td>500</td>
<td>0.55</td>
</tr>
<tr>
<td>400</td>
<td>0.52</td>
</tr>
<tr>
<td>300</td>
<td>0.56</td>
</tr>
<tr>
<td>200</td>
<td>1.15</td>
</tr>
<tr>
<td>100</td>
<td>1.5</td>
</tr>
<tr>
<td>0</td>
<td>1.05</td>
</tr>
</tbody>
</table>

---

*Z.X. Zhang, R.L. Kushwaha*