Introduction

Experimental field studies carried out by several investigators in the past 30 years, e.g., Crory and Reed (1965), Penner (1970, 1974), Penner and Irwin (1969), Penner and Gold (1971), Dalmatov et al. (1973), Domaschuk (1982), Fukuda and Kinosita (1985), Johnson and Esch (1985), Johnson and Buska (1988), Tong et al. (1988), to mention only few, have led to a number of general conclusions on the magnitude and the development of frost heaving stresses acting on a fixed pile, passing through a freezing, frost-susceptible soil. These can be summarized as follows:

A. The main factors affecting the mobilized adfreeze shear stress at a given point of the pile within the freezing zone, are: (1) the soil temperature at that level, (2) the displacement rate at the pile-soil interface, (3) the character of the interface, and (4) the amount of displacement at the interface, producing either slip or non-slip conditions.

B. As a result of the combined action of these various factors, the field observations show that, in general, the peak values of the mobilized adfreeze stress occur during the early freezing period, when heaving rates are high, but maximum uplift forces often occur near the time of maximum frost penetration, late in the winter season, when heaving rates are low.

C. Additional observations include (1) heave forces respond strongly, but with some time lag, to the ground surface temperature variations during the freezing season (Crory and Reed, 1965); (2) heave forces continue increasing even after the frost has attained the permafrost table (Crory and Reed, 1965); (3) there is a size effect present in the mobilized adfreeze strength magnitude, making it inversely proportional to the pile diameter (Penner, 1974).

It is recognized that the development of frost heaving forces on a pile is a complex process, which is not easy to simulate. Although some valuable attempts to simulate it have been made in the past by several investigators (e.g., Penner and Gold, 1971; Dalmatov et al., 1973; Fukuda and Kinosita, 1985; Johnson and Buska, 1988), it is considered that its proper treatment has not yet been achieved. This paper proposes a new method showing how, with some simplifying but plausible assumptions, the magnitude and the development of adfreeze frost heaving forces acting on a fixed pile, combining the effects of frost penetration rate, heave rate and soil temperature at any depth along the pile and at any time, can be calculated in a closed form.

Theory

In order to find a closed-form solution of this complex problem, the following assumptions were made in the development of the theory:

1. The seasonal temperature variation at the ground surface is sinusoidal;

2. The freezing front penetration can be computed by the Modified Berggren formula;
3. The surface heave in situ is approximately proportional to the thickness of the frozen layer;

4. During frost penetration, the displacement and the displacement rate of the heaving soil within the freezing zone along the pile-soil interface decrease linearly with depth;

5. The temperature variation along the pile within the freezing zone is approximately linear;

6. The mobilized adfreeze stress at any point along the pile can be determined if the values of the displacement rate and temperature are known at that point by using the Johnston and Ladanyi (1972) Rod Anchor Theory.

Seasonal temperature variation is assumed to follow a sinusoidal curve, given by:

\[ T_s = T_{ms} - A_s \sin\left(\frac{2\pi t}{p}\right) \]  \[1\]

where: \( T_s \) is temperature at the ground surface (°C); \( T_{ms} \) is mean annual ground surface temperature (°C); \( A_s \) is surface temperature amplitude (°C); \( p \) is period (365 days); \( t \) is time (days).

If only the freezing portion of the sinusoidal curve is considered, with the origin of time at the point of inflection \( (T_{s} = T_{ms}) \), the start of freezing season, \( t_{f,o} \), is obtained by putting \( T_{s} = T_{ms} \) into (1):

\[ t_{f,o} = \left(\frac{p}{2\pi}\right) \arcsin \left(\frac{T_{ms}}{A_s}\right) \]  \[2\]

The end of freezing season is then:

\[ t_{f,end} = (\frac{p}{2}) - t_{f,o} \]

Freezing index. For the assumed sinusoidal surface temperature variation, the cumulative freezing index is obtained by integrating [1] between \( t_{f,o} \) and \( t_{f,end} \), giving:

\[ I_f(t) = T_{ms} (t - t_{f,o}) + A_s \left(\frac{p}{2\pi}\right) \left[ \cos(\frac{2\pi t}{p}) - \cos(\frac{2\pi t_{f,o}}{p}) \right] \]  \[3\]

The freezing index accumulation rate is then

\[ I'_f(t) = T_{ms} - A_s \sin \left(\frac{2\pi t}{p}\right) \]  \[4\]

Frost penetration. The depth of frost penetration at a given time, \( x_0(t) \), can be computed by the Modified Berggren formula

\[ x_0(t) = \frac{\omega}{\sqrt{I_f(t)}} \]  \[5\]

\[ \omega = 60\lambda \left(\frac{48 k_{av}}{L}\right)^{1/2} \]  \[6\]

with: \( k_{av} \) is the average soil thermal conductivity \( 1/2(k_f + k_{th}) \) (W/mK); \( L \) is the latent heat of fusion (MJ/m³); \( \lambda \) is the correction coefficient in the Modified Berggren Equation (Andersland and Ladanyi, 1984, p. 71).

The rate of freezing front penetration is then:

\[ x'_o(t) = (\frac{\omega}{2}) I_f^{-1/2} - I'_f \]  \[7\]

Frost heave and heave rate. Based on field observations quoted by Saarelainen (1992), the surface heave, \( s_s \), is approximately proportional to the thickness of the freezing layer: \( s_s(t) \approx K_xx_o(t) \), and the surface heave rate is then: \( s'_s(t) \approx K_x'x'_o(t) \), where \( K \) is the coefficient of proportionality, depending essentially on the soil frost-susceptibility and the depth of groundwater level. As for the soil cumulative heave and heave rate, it is assumed that they decrease linearly with depth, being zero at the frost front and maximum at the ground surface:

\[ s'_i(t) = s'_s(t)\left[1 - \frac{x_i}{x_o(t)}\right] \]  \[8\]

Tangential uplift stresses at the pile-soil interface. According to Johnston and Ladanyi (1972), if the relative displacement rate at the pile-soil interface at depth \( x_i \) is \( s_i \), then the corresponding mobilized tangential stress at that depth, \( \tau_{a,i} \), can be computed from

\[ \tau_{a,i} = \tau_{c,\theta} \left[\frac{(n - 1)}{\gamma_c} \right]^{1/2} \left[\frac{S_i(t)}{a} \right] \]  \[9\]

where: \( a \) is the pile radius; \( n \) is the creep exponent; \( \gamma_c \) is an arbitrary reference shear strain rate; \( \tau_{c,\theta} \) is the shear creep modulus, related to the general creep modulus, \( \sigma_{co} \), and the temperature through

\[ \tau_{c,\theta} = \sigma_{co} \left(1 + \frac{\theta}{\theta_o}\right)^w \]  \[10\]

with, \( \theta \) is the absolute value of the negative temperature; \( \theta_o \) is 1°C, and \( w \) is the experimental temperature exponent. Some typical values of the three above parameters for various soils can be found, e.g., in Andersland and Ladanyi (1994). In addition, \( \tau_{a,i} \) is also affected by the pile type and the state of the pile-soil interface. According to Weaver and Morgenstern (1981), the maximum \( \tau_{a,i} \) value, such as in Equation (9), would be valid only for corrugated steel pipe piles, while this value should be multiplied by 0.7 for smooth concrete piles, and by 0.6 for smooth steel piles.

Temperature variation in the freezing layer along the pile can be computed by rigorous methods, but, according to the field observations (e.g., Penner 1974, Figures 8 and 9; Johnson and Esch, 1985, Figure 7), it can be closely approximated by a linear relationship:
Numerical simulation

In order to show the kind of predictions this theory is able to furnish, it was decided to apply it to two different sites where numerous pile heave force observations have been made. The first one is in the Ottawa, Ontario, area, where some early observations were made by Penner and his collaborators in the 1970's (Penner and Gold, 1971; Penner, 1974), and the second one is located near Fairbanks, Alaska, used by Crory and Reed (1965), Johnson and Esch (1985) and Johnson and Buska (1988).

For each of the two sites, climatic conditions were determined, and the computation was made assuming a fixed smooth steel pile of 0.15 m diameter, embedded in a frost-susceptible ice-rich silt or silty clay. For illustration purposes, the computation of uplift stresses and forces was made for two cases: (1) assuming no slip at the interface, and (2) assuming that slip at the interface occurs when relative displacement attains 2 cm (as found in Johnston and Ladanyi (1972) rod anchor tests), resulting in 50% reduction of the available adfreeze strength on a portion of pile length. In fact, this limiting slip displacement is not a constant, but depends on the pile interface roughness, the pile material, and the method of pile installation.

Finally, the solution of the problem of evolution of uplift forces acting on a rigid fixed pile is obtained by the following procedure: (1) at selected times within the freezing period, find frost penetration and frost penetration rate, surface heave and heave rate, relative displacement and displacement rate at selected depths along the pile; (2) using Equation (9), determine the mobilized shear stress at each depth, taking into account the soil temperature; (3) the total uplift force $T$ acting on the pile is then:

$$ T = p \sum_{x_i=0}^{x_f=x_0} t_{a,i} \Delta x_i $$

\[ \text{[12]} \]

Climatic data. The following average climatic data were used for the two sites:

**Ottawa:**
- $I_f = -482.7^\circ$Cdays, $I_{th} = 3206.2^\circ$Cdays, $T_{ms} = 6.1^\circ$C, $A_s = 18^\circ$C,
- The freezing season ($t_{fo}$) starts on average on December 15 each year.

**Fairbanks:**
- $I_f = -3144.8^\circ$Cdays, $I_{th} = 2049.8^\circ$Cdays, $T_{ms} = -3^\circ$C, $A_s = 23^\circ$C.

The freezing season ($t_{fo}$) starts on average on October 1 each year.

**Thermal properties:** For both sites:
- Assuming: total water content = 30%; dry density = 1300 kg/m$^3$; unfrozen water content = 5%;
- Thermal conductivities: $k_f = 1.7$ W/m$^\circ$K, $k_{th} = 1.1$ W/m$^\circ$K, $k_{av} = 1.4$ W/m$^\circ$K.
- Heat capacities: $c_{vf} = 1.878$ MJ/m$^3$K, $c_{vth} = 2.558$ MJ/m$^3$K, $c_{vav} = 2.218$ MJ/m$^3$K.

**Frost penetration:** According to the modified Berggren formula, one gets (in m and days):
- For Ottawa: $x_f(t) = 0.03918 [I_f(t)]^{1/2}$
- For Fairbanks: $x_f(t) = 0.04202 [I_f(t)]^{1/2}$

**Creep properties:** They were assumed as for ice rich silt in Table 5.3 in Andersland and Ladanyi (1994):
- Creep modulus: $\sigma_{co} = 103$ kPa for strain
  - rate $\varepsilon'_c = 10^{-5}$ h$^{-1}$,
  - Creep exponent: $n = 3$; Temperature exponent:
    - $w = 0.37$, and $\theta_o = 1^\circ$C. One gets:
    - $\gamma'_c = 9 \times 10^{-5}$ h$^{-1}$, and Equation (10) yields for $x = x_i$
      $\tau_{a,i} = 2895.8 (1 + \theta_i)^{0.37} (s'_i / a)^{1/3}$ [13]

in kPa and hours. However, for a smooth steel pipe pile, according to Weaver and Morgenstern (1981), this value should be multiplied by 0.6, so that the numerical factor in Eq.(13) becomes: 2895.8 $\times$ 0.6 = 1737.5. This latter numerical factor was used in all computations in this paper.

Results of simulation

Because of space limitations, only a portion of the results of the computations can be shown here. Figures 1 to 6 show for the two sites, Ottawa and Fairbanks, and for both cases, slip and non-slip, respectively:
- Daily variation of temperature and cumulative freezing index, Figures 1 and 4;
- Heave rate at the ground surface (which reduces to zero at the freezing front), Figures 2(a) and 5(a);
- Time variation of the average uplift shear stress acting on the pile, Figures 2(b) and 5(b);
- Variation of mobilized uplift shear stresses, as a function of frost penetration, Figures 3(a) and 6(a);
- Time variation of the total heave force acting on a fixed pile of 0.15 m diameter, Figures 3(b) and 6(b);

Although this exercise did not allow a first class type prediction, because the field investigations were made
Figure 1. Ottawa site:
(a) Daily temperature variation;
(b) Daily freezing index variation.
Figure 2. Ottawa site:
(a) Time variation of surface heave rate; 
(b) Time variation of the average uplift shear stress acting on the pile.
Figure 3. Ottawa site:
(a) Time variation of uplift shear stresses along the pile;
(b) Time variation of the total uplift heave force acting on a steel pile of 0.15 m diam.
Figure 4. Fairbanks site:
(a) Daily temperature variation;
(b) Daily freezing index variation.

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Figure 5. Fairbanks site:
(a) Time variation of surface heave rate;
(b) Time variation of the average uplift shear stress acting on the pile.
Figure 6. Fairbanks site:
(a) Time variation of uplift shear stresses along the pile;
(b) Time variation of the total uplift heave force acting on a steel pile of 0.14 m diam.
by other investigators in the past, it has nevertheless yielded results qualitatively and quantitatively close to the observations. In particular, it is found that:

(1) The shape of heave force variation with time (Figures 3(b) and 6(b)) agrees with the observations in the quoted field tests. As an example, Figure 7 shows the results of measurement of total uplift force and average adfreeze stress carried out by Penner (1974) on a 12.75 in. (0.324 m) diameter steel pile embedded in Leda clay at a depth of 5 ft (1.50 m).

(2) The magnitude of maximum average frost heaving stresses agrees well with the observed values:

For Ottawa site: Predicted: 180 kPa with slip, 230 kPa, with no-slip, to be compared with 100 kPa to 240 kPa found by Penner and Gold (1971) and Penner (1974), (quoted by Johnston, 1981).

For Fairbanks site: Predicted: 210 kPa with slip, 320 kPa no-slip, which compares well with 275 kPa (Crory and Reed, 1965), and 324 kPa (Johnson and Esch, 1985).

(3) Mobilized adfreeze strength is approximately proportional to the heave rate, as shown by Penner (1974, Fig.13). (It is actually theoretically proportional to \( s^{1/3} \), according to Equation (13)).

(4) The theory (Equation 9) explains the source of the size effect, found by Penner (1974).

Conclusions

The approximate closed-form solution for calculating the magnitude and the development of adfreeze frost heaving stresses acting on a pile, shows the importance of taking into account simultaneously the effects of frost penetration rate, heave rate, and time-variation of soil temperature along the pile. Although this exercise did not allow a first class type prediction, because the field investigations were made by other investigators in the past, it has nevertheless yielded results qualitatively and quantitatively close to the observations. In particular, it is found that:

(1) The shape of heave force variation with time (Figures 3(b) and 6(b)) agrees with the observations in the quoted field tests (Figure 7).

(2) The magnitude of the average frost heaving stresses agrees well with the observed values. It is found that the no-slip assumption with the reduction factor due to pile type gives reasonable prediction results.
(3) The mobilized adfreeze strength is approximately proportional to the heave rate, as shown by Penner (1974).

(4) The theory (Equation 9) explains the source of the size effect, found by Penner (1974).

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References


